Escola Secundária/3 da Sé-Lamego

Ficha de Trabalho de Matemática

Ano Lectivo 2003/04 ***Relações entre razões trigonométricas; equações*** **11.º Ano**

Nome: _____ N.º: ___ Turma: ___

1. Simplifique as expressões:

a) $\cos(\pi-\alpha)+sen(\frac{\pi}{2}+\alpha)$

b) $sen(\frac{3\pi}{2}-\alpha)+2\cos(\pi+\alpha)$

c) $2sen(\pi+\alpha)-\cos(\frac{3\pi}{2}+\alpha)+sen(-\alpha)$

d) $tg(\pi-\alpha)+tg(-\alpha)$

e) $\cos(x-\pi)-\cos(3\pi-x)+sen(-\frac{5\pi}{2}+x)$

f) $\cos(-x)+sen(\pi-x)+\cos(\pi+x)$

g) $sen(x-5\pi)+\cos(\frac{3\pi}{2}+x)-sen(\frac{\pi}{2}-x)$

h) $sen(x-\frac{7\pi}{2})+\cos(5\pi-x)-\cos(-\frac{7\pi}{2}+x)$

2. Calcule o valor exacto de:

a) $sen\frac{\pi}{6}+3\cos\frac{\pi}{3}$

b) $tg\frac{\pi}{3}-2sen\frac{2\pi}{3}$

c) $2sen(-\frac{3\pi}{4})+\cos\frac{7\pi}{4}-tg\frac{\pi}{4}$

d) $\cos 750^\circ - 2sen\,1140^\circ + tg(-405^\circ)$

e) $sen\,1470^\circ - \cos 1080^\circ - 2tg\,765^\circ$

f) $sen(-\frac{7}{6}\pi)-sen(\frac{17}{6}\pi)+\cos(-\frac{2}{3}\pi)+tg\frac{8}{3}\pi$

3. Através de uma construção geométrica represente:

a) Um ângulo do 2.º Q de seno 0,25.

b) Um ângulo de 1.º Q de seno 1,5.

c) Um ângulo do 3.º Q de co-seno –0,75.

d) Um ângulo de tangente –2.

4. Usando fórmulas de trigonometria, calcule:

a) $tg\,\beta$, sendo $sen\,\beta=-\frac{1}{2}$ e β do 4.º Q.

b) $sen\,x-\cos(\pi-x)$, sendo $\cos x=-\frac{\sqrt{3}}{2}$.

5. Sabendo que x é um ângulo do 3.º quadrante e que $tg\,x=\frac{1}{4}$, calcule:

a) $\cos(\pi-x)$

b) $\frac{1}{tg(\pi+x)}$

c) $sen(2\pi-x)$

d) $\frac{1}{tg(\frac{\pi}{2}+x)}$

6. Acerca do ângulo α, sabe-se que $sen\,(\pi+\alpha)=-\frac{\sqrt{2}}{3} \;\wedge\; 0<\alpha<\frac{\pi}{2}$. Calcule $\cos(\pi-\alpha)+tg\,(\pi+\alpha)$.

7. Resolva as equações trigonométricas:

a) $sen\,(2x+10^\circ)=sen\,50^\circ$

b) $sen\,(2x)=-sen\,x$

c) $\cos(x-\frac{\pi}{4})=\frac{1}{2}tg\,\frac{\pi}{3}$

d) $\cos(2x)=-\cos\frac{\pi}{5}$

e) $2\cos x=-\sqrt{3}$

f) $sen\,x=0{,}5$

g) $tg\,(x+15^\circ)=-\sqrt{3}$

h) $tg\,(2x)=-tg\,\frac{\pi}{6}$

i) $sen\,x.\cos x=0$

j) $\cos^2 x-\text{sen}^2\,x=0$

8. Resolva cada uma das seguintes equações trigonométricas e indique para cada uma delas as soluções que pertencem ao intervalo $[-\pi,\pi]$.

a) $1+2\cos x=0$. **b)** $3sen\,(2x)=-4$. **c)** $sen\,(3x)=-sen\,x$. **d)** $\frac{1}{\sqrt{3}}+tg\,x=0$.

SOLUÇÕES

1.

a) 0

b) $-3\cos\alpha$

c) $-4\,\text{sen}\,\alpha$

d) $-2tg\alpha$

e) $-\cos x$

f) $\text{sen}\,x$

g) $-\cos x$

h) $\text{sen}\,x$

2.

a) 2

b) 0

c) $-\frac{2+\sqrt{2}}{2}$

d) $-\frac{2+\sqrt{3}}{2}$

e) $-\frac{5}{2}$

f) $-\frac{1}{2}$

4.

a) $-\frac{\sqrt{3}}{3}$

b) $-\frac{\sqrt{3}\mp 1}{2}$

5.

a) $\frac{4\sqrt{17}}{17}$

b) 4

c) $\frac{\sqrt{17}}{17}$

d) $-\frac{1}{4}$

6. $\frac{3\sqrt{14}-7\sqrt{7}}{21}$

7.

a) $x=20^\circ+k.180^\circ \;\vee\; x=60^\circ+k.180^\circ \;,k\in Z$

b) $x=\frac{2k\pi}{3} \;\vee\; x=\pi+2k\pi \;,k\in Z$

c) $x=\frac{5\pi}{12}+2k\pi \;\vee\; x=\frac{\pi}{12}+2k\pi \;,k\in Z$

d) $x=\pm\frac{3\pi}{5}+k\pi \;,k\in Z$

e) $x=\pm\frac{5\pi}{6}+2k\pi \;,k\in Z$

f) $x=\frac{\pi}{6}+2k\pi \;\vee\; x=\frac{5\pi}{6}+2k\pi,\; k\in Z$

g) $x=105^\circ+k.180^\circ \;,k\in Z$

h) $x=\frac{5\pi}{12}+\frac{k\pi}{2} \;,k\in Z$

i) $x=\frac{k\pi}{2} \;,k\in Z$

j) $x=\frac{\pi}{4}+\frac{k\pi}{2} \;,k\in Z$

8.

a) $S=\{-\frac{2\pi}{3},\frac{2\pi}{3}\}$

b) $S=\varnothing$ (A equação é impossível)

c) $S=\{-\pi,-\frac{\pi}{2},0,\frac{\pi}{2},\pi\}$

d) $S=\{-\frac{\pi}{6},\frac{5\pi}{6}\}$

O Professor

Proposta de Resolução:

1.

a) $\cos(\pi-\alpha)+sen(\frac{\pi}{2}+\alpha)=-\cos\alpha+\cos(-\alpha)=-\cos\alpha+\cos\alpha=0$

b) $sen(\frac{3\pi}{2}-\alpha)+2\cos(\pi+\alpha)=sen(\pi+\frac{\pi}{2}-\alpha)-2\cos\alpha=-sen(\frac{\pi}{2}-\alpha)-2\cos\alpha=-\cos\alpha-2\cos\alpha=-3\cos\alpha$

c) $2sen(\pi+\alpha)-\cos(\frac{3\pi}{2}+\alpha)+sen(-\alpha)=-2sen\,\alpha-\cos(\pi+\frac{3\pi}{2}+\alpha)-sen\,\alpha=-3sen\,\alpha+sen(-\alpha)=-4sen\,\alpha$

d) $tg(\pi-\alpha)+tg(-\alpha)=tg(-\alpha)-tg\,\alpha=-2tg\,\alpha$

e) $\cos(x-\pi)-\cos(3\pi-x)+sen(-\frac{5\pi}{2}+x)=-\cos x+\cos x+sen(-\frac{\pi}{2}+x)=-sen(\frac{\pi}{2}-x)=-\cos x$

f) $\cos(-x)+sen(\pi-x)+\cos(\pi+x)=\cos x+sen\,x-\cos x=sen\,x$

g) $sen(x-5\pi)+\cos(\frac{3\pi}{2}+x)-sen(\frac{\pi}{2}-x)=-sen\,x-\cos(\frac{\pi}{2}+x)-\cos x=-sen\,x-sen(-x)-\cos x=-\cos x$

h) $sen(x-\frac{7\pi}{2})+\cos(5\pi-x)-\cos(-\frac{7\pi}{2}+x)=sen(\frac{\pi}{2}+x)-\cos x-\cos(\frac{\pi}{2}+x)=\cos(-x)-\cos x-sen(-x)=sen\,x$

2.

a) $sen\frac{\pi}{6}+3\cos\frac{\pi}{3}=\frac{1}{2}+3\times\frac{1}{2}=2$

b) $tg\frac{\pi}{3}-2sen\frac{2\pi}{3}=\sqrt{3}-2\times\frac{\sqrt{3}}{2}=0$

c) $2sen(-\frac{3\pi}{4})+\cos\frac{7\pi}{4}-tg\frac{\pi}{4}=2\times(-\frac{\sqrt{2}}{2})+\frac{\sqrt{2}}{2}-1=-\frac{2+\sqrt{2}}{2}$

d) $\cos 750^\circ-2sen\,1140^\circ+tg(-405^\circ)=\cos 30^\circ-2sen\,60^\circ+tg(-45^\circ)=\frac{\sqrt{3}}{2}-2\times\frac{\sqrt{3}}{2}-1=-\frac{2+\sqrt{3}}{2}$

e) $sen\,1470^\circ-\cos 1080^\circ-2tg\,765^\circ=sen\,30^\circ-\cos\,0^\circ-2tg\,45^\circ=\frac{1}{2}-1-2=-\frac{5}{2}$

f) $sen(-\frac{7}{6}\pi)-sen(\frac{17}{6}\pi)+\cos(-\frac{2}{3}\pi)+tg\frac{8}{3}\pi=\frac{1}{2}-\frac{1}{2}-\frac{1}{2}=-\frac{1}{2}$

4.

a) Como $\beta\in 4.^\circ Q$, então $\cos\beta=+\sqrt{1-(-\frac{1}{2})^2}=\frac{\sqrt{3}}{2}$. Logo, $tg\,\beta=\frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}=-\frac{\sqrt{3}}{3}$.

b) Como $\cos x=-\frac{\sqrt{3}}{2}$, então $sen\,x=\pm\sqrt{1-(-\frac{\sqrt{3}}{2})^2}=\pm\frac{1}{2}$.

Logo, $sen\,x-\cos(\pi-x)=sen\,x+\cos x=\pm\frac{1}{2}-\frac{\sqrt{3}}{2}=-\frac{\sqrt{3}\mp 1}{2}$.

5. Como $x \in 3.^{\circ}Q$ e $1+tg^2 x = \dfrac{1}{\cos^2 x}$, então $\cos x = -\sqrt{\dfrac{1}{1+\frac{1}{16}}} = -\dfrac{4\sqrt{17}}{17}$. Logo, $sen\ x = -\sqrt{1-\frac{16}{17}} = -\dfrac{\sqrt{17}}{17}$.

a) Logo, $\cos(\pi - x) = -\cos x = \dfrac{4\sqrt{17}}{17}$.

b) Logo, $\dfrac{1}{tg\ (\pi + x)} = \dfrac{1}{tg\ x} = \dfrac{1}{\frac{1}{4}} = 4$.

c) Logo, $sen\ (2\pi - x) = -sen\ x = \dfrac{\sqrt{17}}{17}$.

d) Logo, $\dfrac{1}{tg\ (\frac{\pi}{2}+x)} = \dfrac{1}{\frac{sen\ (\frac{\pi}{2}+x)}{\cos(\frac{\pi}{2}+x)}} = \dfrac{\cos(\frac{\pi}{2}+x)}{sen\ (\frac{\pi}{2}+x)} = \dfrac{-sen\ x}{\cos x} = -\dfrac{1}{4}$.

6. Ora, $sen\ (\pi+\alpha) = -\dfrac{\sqrt{2}}{3} \ \wedge\ 0 < \alpha < \dfrac{\pi}{2} \Leftrightarrow sen\ \alpha = \dfrac{\sqrt{2}}{3} \ \wedge\ 0 < \alpha < \dfrac{\pi}{2}$. Logo, $\cos\alpha = +\sqrt{1-(\frac{\sqrt{2}}{3})^2} = \dfrac{\sqrt{7}}{3}$.

Pelo que $\cos(\pi-\alpha) + tg\ (\pi+\alpha) = -\cos\alpha + tg\ \alpha = -\dfrac{\sqrt{7}}{3} + \dfrac{\frac{\sqrt{2}}{3}}{\frac{\sqrt{7}}{3}} = -\dfrac{\sqrt{7}}{3} + \dfrac{\sqrt{2}}{\sqrt{7}} = -\dfrac{\sqrt{7}}{3} + \dfrac{\sqrt{14}}{7} = \dfrac{3\sqrt{14}-7\sqrt{7}}{21}$.

7.

a)

$$sen\ (2x+10^{\circ}) = sen\ 50^{\circ} \Leftrightarrow 2x+10^{\circ} = 50^{\circ}+k\times 360^{\circ} \ \vee\ 2x+10^{\circ} = 130^{\circ}+k\times 360^{\circ},\ k\in Z$$
$$\Leftrightarrow x = 20^{\circ}+k\times 180^{\circ} \ \vee\ x = 70^{\circ}+k\times 180^{\circ},\ k\in Z$$

b)

$$sen\ (2x) = -sen\ x \Leftrightarrow sen\ (2x) = sen\ (-x)$$
$$\Leftrightarrow 2x = -x+2k\pi \ \vee\ 2x = \pi + x + 2k\pi,\ k\in Z$$
$$\Leftrightarrow x = \frac{2k\pi}{3} \ \vee\ x = \pi + 2k\pi,\ k\in Z$$

c)

$$\cos(x-\frac{\pi}{4}) = \frac{1}{2}tg\ \frac{\pi}{3} \Leftrightarrow \cos(x-\frac{\pi}{4}) = \frac{\sqrt{3}}{2}$$
$$\Leftrightarrow x-\frac{\pi}{4} = \pm\frac{\pi}{6}+2k\pi,\ k\in Z$$
$$\Leftrightarrow x = \frac{5\pi}{12}+2k\pi \ \vee\ x = \frac{\pi}{12}+2k\pi,\ k\in Z$$

d)

$$\cos(2x) = -\cos\frac{\pi}{5} \Leftrightarrow \cos(2x) = \cos(\pi+\frac{\pi}{5})$$
$$\Leftrightarrow 2x = \pm\frac{6\pi}{5}+2k\pi,\ k\in Z$$
$$\Leftrightarrow x = \pm\frac{3\pi}{5}+k\pi,\ k\in Z$$

e)

$$2\cos x = -\sqrt{3} \Leftrightarrow \cos x = -\frac{\sqrt{3}}{2}$$
$$\Leftrightarrow x = \pm\frac{5\pi}{6}+2k\pi,\ k\in Z$$

f)

$$sen\ x = 0{,}5 \Leftrightarrow x = \frac{\pi}{6} + 2k\pi \ \vee \ x = \pi - \frac{\pi}{6} + 2k\pi,\ k \in Z$$
$$\Leftrightarrow x = \frac{\pi}{6} + 2k\pi \ \vee \ x = \frac{5\pi}{6} + 2k\pi,\ k \in Z$$

g)

$$tg\ (x + 15^\circ) = -\sqrt{3} \Leftrightarrow x + 15^\circ = 120^\circ + k \times 180^\circ,\ k \in Z$$
$$\Leftrightarrow x = 105^\circ + k \times 180^\circ,\ k \in Z$$

h)

$$tg\ (2x) = -tg\ \frac{\pi}{6} \Leftrightarrow tg\ (2x) = tg\ (-\frac{\pi}{6})$$
$$\Leftrightarrow 2x = -\frac{\pi}{6} + k\pi,\ k \in Z$$
$$\Leftrightarrow x = -\frac{\pi}{12} + \frac{k\pi}{2},\ k \in Z$$
$$\Leftrightarrow x = \frac{5\pi}{12} + \frac{k\pi}{2},\ k \in Z$$

i)

$$sen\ x.\cos x = 0 \Leftrightarrow sen\ x = 0 \ \vee \ \cos x = 0$$
$$\Leftrightarrow x = \frac{k\pi}{2},\ k \in Z$$

j)

$$\cos^2 x - sen^2 x = 0 \Leftrightarrow \cos^2 x - (1 - \cos^2 x) = 0$$
$$\Leftrightarrow \cos^2 x = \frac{1}{2}$$
$$\Leftrightarrow \cos x = \pm\frac{\sqrt{2}}{2}$$
$$\Leftrightarrow x = \frac{\pi}{4} + \frac{k\pi}{2},\ k \in Z$$

8.

a) Ora,

$$1 + 2\cos x = 0 \Leftrightarrow \cos x = -\frac{1}{2}$$
$$\Leftrightarrow x = \pm\frac{2\pi}{3} + 2k\pi,\ k \in Z$$

Logo, o conjunto das soluções no intervalo $[-\pi, \pi]$ é: $S = \{-\frac{2\pi}{3}, \frac{2\pi}{3}\}$.

b) Ora,

$$3sen\ (2x) = -4 \Leftrightarrow sen\ (3x) = -\frac{4}{3}$$
$$\Leftrightarrow x \in \varnothing$$

Logo, o conjunto das soluções no intervalo $[-\pi, \pi]$ é: $S = \varnothing$.

c) Ora,

$$sen\ (3x) = -sen\ x \Leftrightarrow sen\ (3x) = sen\ (-x)$$
$$\Leftrightarrow 3x = -x + 2k\pi \ \vee \ 3x = \pi + x + 2k\pi,\ k \in Z$$
$$\Leftrightarrow x = \frac{k\pi}{2} \ \vee \ x = \frac{\pi}{2} + k\pi,\ k \in Z$$

Logo, o conjunto das soluções no intervalo $[-\pi, \pi]$ é: $S = \{-\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi\}$.

d) Ora,

$$\frac{1}{\sqrt{3}} + tg\ x = 0 \Leftrightarrow tg\ x = -\frac{\sqrt{3}}{3}$$
$$\Leftrightarrow x = -\frac{\pi}{6} + k\pi,\ k \in Z$$

Logo, o conjunto das soluções no intervalo $[-\pi, \pi]$ é: $S = \{-\frac{\pi}{6}, \frac{5\pi}{6}\}$.